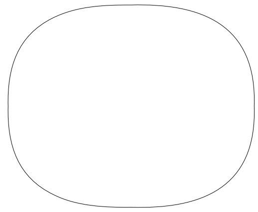
Superellipses…. Lamé Curves where R>2



To an artist, curved shapes can be interesting and beautiful. To an engineer or designer, they have an even greater importance—it is critical to be able to guide tools and computer aided design programs exactly, so the mathematical definition of a particular curve must be known and manipulated. Ellipses, circles, rectangles and squares are everywhere in designs. More rarely we see parabolas, catenaries, ovoids and a few other named curves such as Lamé curves—an important class of curves with the simple equation:

$$\left|\frac{X}{A}\right|^{R}+\left|\frac{Y}{B}\right|^{R}=1$$

Although all Lamé curves find their uses, the Cartesian curves where R>2 are called Superellipses. Piet Hein limited his design curves to R=2.5, probably because he started using them before the age of the personal computer. Since the shape approaches a rectangle quite rapidly as exponent R increases, Superellipses of R>10 aren't much different from rectangles.

Piet Hein actually owns the copyright to the name "Superellipse" and popularized the idea of using these curves in design. These curves are useful because they are compelling closed simple shapes, like ellipses and ovoids, while possessing elegant and useful characteristics.

Superellipses have found new life since computerized numerically controlled CNC mills have become the usual method of cutting special curves.

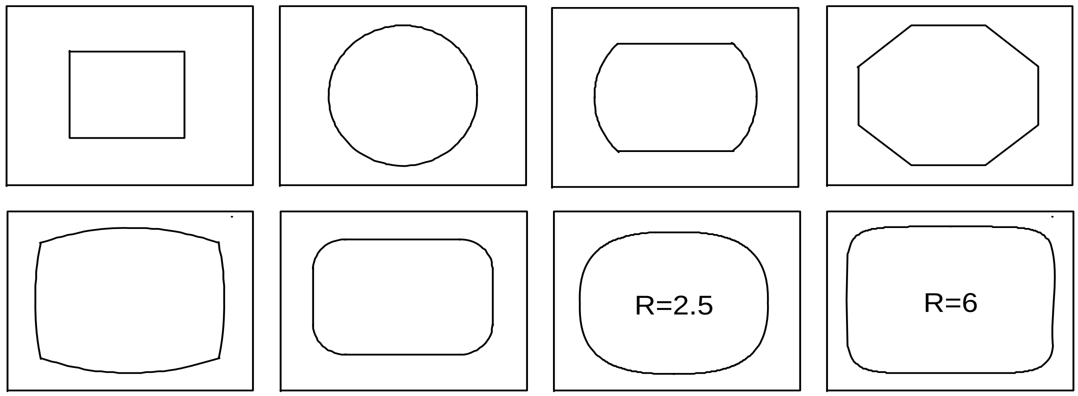
So how is a superellipse useful?

1) It can elegantly substitute for a rectangle with corner radii, with the further advantage of eliminating flat sides. Flat sides can be a design problem either for aesthetic or structural reasons.

2) It can advantageously replace a circle or ellipse where increasing the internal area (and of course, extruded volume) is required.

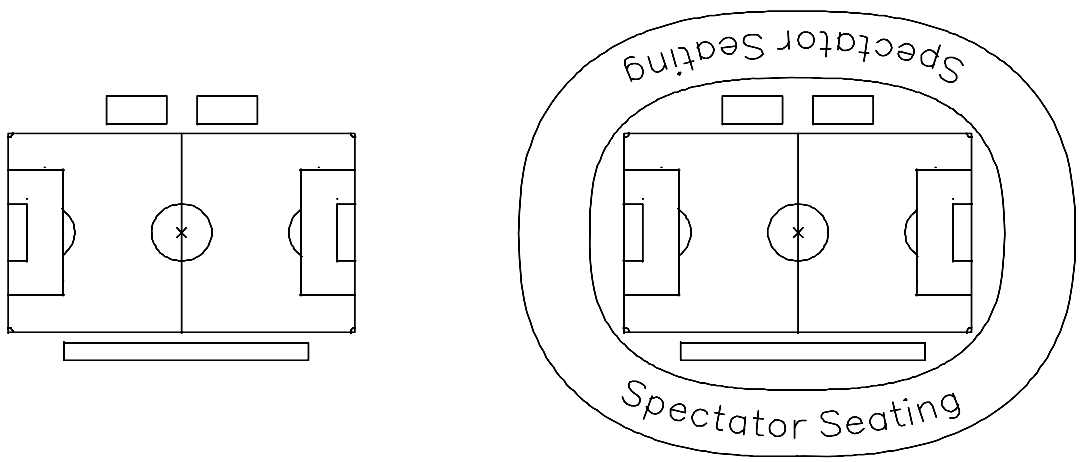
3) It is a well-defined and easy-to-use shape where hard-to-define blending and smoothing would otherwise be needed to remove corners.

Much inspiration and history can be found at Piet Hein's website www.PietHein.com , as well as the 1965 Scientific American article on the subject. Here are some problem and solution examples:



A Short History of Television Screen Shapes

Historically TV screens were compromises between the vacuum display tube that was more practical to make with curved shapes, and rectangular movie film frames. The superelliptical shape of the TV screen (the last two with Lamé exponents shown) is surprising if only because it is so natural that nobody thinks much about it anymore. Other shapes disappeared quickly. Indeed, usually only TV historians are familiar with the earlier shapes.

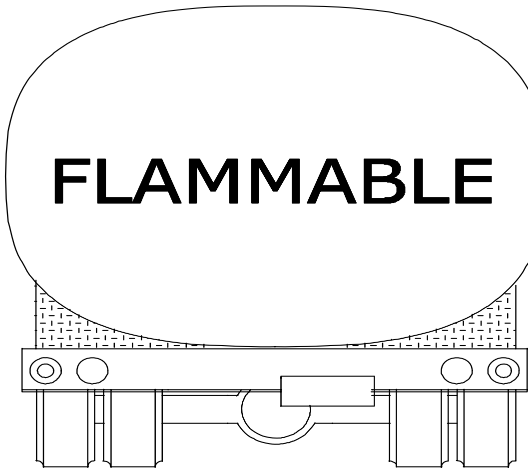


Here is an example of a soccer stadium showing the rectangular field and the elegant use of superelliptical spectator seating. Certainly as elegant a solution as can be found.

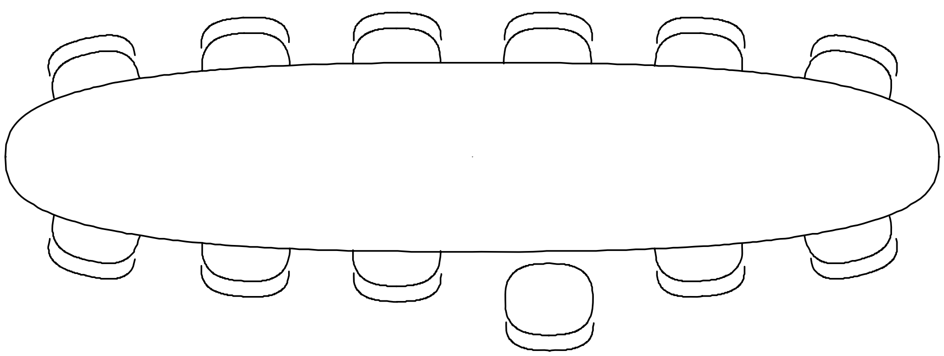
Almost the inverted problem—The maximum size Ice skating rink in a rectangular building has been done too. Nobody uses the corners anyway.

Piet Hein achieved fame when he invented the superellipse to put a roundabout in Sergels torg (Sergel's Square) in Stockholm, Sweden. The space is rectangular, the pool in the middle is elliptical…so….!

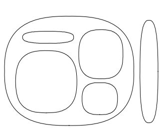
There is one other property of Lame curves that is interesting: Curves of R=0-1 are unstable in equilibrium on both axes. Curves of R=1-2 are unstable in equilibrium in one axis and neutral in the other, but all superellipses are stable in both axes regardless of their aspect ratio—that is they do not roll when pushed slightly. Piet Hein used this interesting property as the basis for his 3D “Super-egg”.



Here is an example of an airport fuel truck. The tanker will still pass under the wing of an aircraft, or a low bridge. The maximum amount of fuel that could be carried on the back of a truck would be in a square or rectangular tank. But that would not be practical because of stresses at the corners and flat sides buckling. If one rounds the corners, one still has flat side that flex, often with disastrous results. Thus round or elliptical cross-sections are often seen. If you have an elliptical tank section you can increase the volume by turning it into a superellipse.

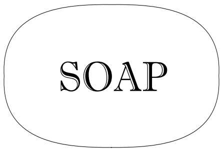


A plan view of a custom superelliptical conference table and chairs.

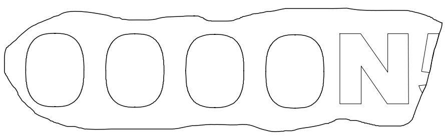


Food service small tray. Perfect for airplanes and crowded places. Consider how this would look with standard round cups and plates—and you will agree this is superior.

An elegant and useful form.

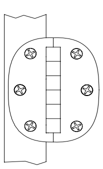


Occasionally someone decides to make soap in some other shape, but it is never a great success. Cylindrical forms are impossible to hold when wet and slippery. Rectangular forms drift towards Superellipses anyway. You might as well start with one.



You've certainly seen these windows on every jetliner. They maximize viewing area while minimizing corner stresses. They are beautiful too.

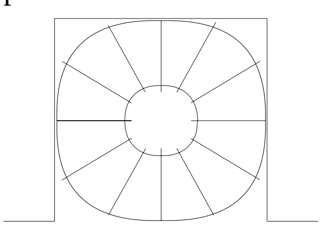
Jewelers call these by various names but they are Superellipses®.



Hinges

Consider how much easier this hinge can be inset into a door edge with machine tools. Most hinges fit in rectangular insets that are hard to make.

Consider how much better a Superelliptical staircase fits into a rectangular space compared to a spiral staircase. Yet this form could be made with only two basic tread planforms.



Here is my version of a Lamé curve generator in Microsoft Excel.

http://www.periheliondesign.com/superellipse_generator.xls
(When Windows ask you to allow macros, click YES.)